Centre for Computational Geostatistics (CCG)
Guidebook Series Vol. 4

Guide to Recoverable Reserves with Uniform Conditioning

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Centre for Computational Geostatistics (CCG)
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Preface

In September 2004 Clayton Deutsch and I decided that uniform conditioning (UC) would make a good reading course as part of my degree requirements. We decided on UC because it is being used by practitioners worldwide and we did not know very much about it. For this reason, the UC meeting at the Geostatistics Congress in Banff was very fortuitous. Many people at the meeting wanted to know the details of how UC works, where it is appropriate, and how they can apply UC. The general idea of the project was to dive into the details of UC and present UC in an easy to understand format. The focus is on the exact procedure for UC; comments on the applicability are limited.

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Chapter 1

Introduction

The calculation of recoverable reserves must consider the application of a cutoff grade and a selective mining unit (SMU) size. The cutoff grade is calculated using economic and management parameters and is considered a known parameter. The size of the SMU is based on the deposit type and the chosen mining equipment. The change of support from the small scale data to the appropriate SMU is often accomplished under a Gaussian model. The uniform conditioning (UC) approach is one Gaussian model. UC is used in the mining industry, yet the literature available on this topic remains limited. The aim of this guide is to explain the details of UC in a concise and complete manner.

There are four main sections in this guide: (1) Literature Review, (2) The Theory of Uniform Conditioning, (3) a Case Study, and (4) Practical Considerations. The literature review section outlines some of the more important literature for uniform conditioning. More detailed information is in the annotated bibliography. The theory section describes the details of uniform conditioning as well as the other required components. Hermite polynomial fitting, the discrete Gaussian model, and the Gaussian anamorphosis are discussed. A small case study describing each step of the UC process is shown in the first case study. This permits readers to become familiar with all of the steps required. A large scale case study shows how UC can be applied to global recoverable reserve estimation. Some of the practical considerations required for UC are discussed in the last section, including the different software available to perform UC and a practical check for the change of support model.

Chapter 2

Literature Review

This chapter provides an overview of the literature regarding uniform conditioning. Most of the publications can be grouped into two categories: (1) those that deal with the theoretical aspects of uniform conditioning, and (2) those that present case studies where uniform conditioning was used. More details can be found in the annotated bibliography.

2.1 Theoretical Literature

The earliest reference to uniform conditioning was Matheron in 1974 [9]. Although I do not have a copy of this myself, it was referenced in some of Remacre's work.

In 1984 Remacre completed his thesis entitled "L'estimation du Récupérable local, le conditionnement uniforme [11]." That same year Guibal and Remacre published a paper on the local estimation of recoverable reserves that included uniform conditioning [5].

Appendix C in Guibal's 1987 paper [4] describes uniform conditioning applied entirely in Gaussian units. Remacre presents a nice summary of uniform conditioning in a paper in the same collection as Guibal's [12].

Rivoirard's book, published in 1994, derives the theory and proofs that form the theoretical basis for uniform conditioning [14]. In addition to uniform conditioning, it contains detailed aspects of the Gaussian anamorphosis, and the change of support models.

A very interesting paper on the history of non-linear geostatistics was presented by Vann and Guibal in 1998 [16]. The place of uniform conditioning is discussed, with no theory. This is a good comparison paper of the different methods availble.

Chilès and Delfiner mention uniform conditioning in their book [2].

Roth and Deraisme presented a paper in 2000 at the geostatistics congress on the information effect and uniform conditioning [15]. They presentedthe methodology and a case study for incorporating the information effect into reserve estimates using uniform conditioning. This is the only paper found that presents a model for the information effect that is consistent with the discrete Gaussian model.

2.2 Case Studies

Almost all of the references that covered the theory of uniform conditioning presented short case studies. These include Remacre and Guibal [5], Guibal [4], and Remacre [12]. Additional papers have been published/presented showing case studies where uniform conditioning has been applied.

Remacre presented a comparative case study between uniform conditioning and indicator kriging at the Avignon Geostatistics Congress [13] Assibey-Bonsu and Krige presented a paper at the 1999 APCOM comparing several different methods for estimating recoverable resources [1].

Chapter 3

Theory of Uniform Conditioning

Uniform conditioning is used for estimating the recoverable reserves inside a mining panel using the estimated panel grade and a change of support model. We make the assumption that if the panel grade is known, then the distribution of the smu's within that panel is also known. Figure 3.1 shows a schematic where uniform conditioning is applied. The name uniform conditioning comes from the estimate of the recoverabe reserves is conditioned to one variable, the panel grade.

The workflow for uniform conditioning (UC) can be broken down into 6 steps (adapted from [12]):

1. Estimate the panel grades.
2. Fit the Discrete Gaussian Model (DGM) to the data.
3. Determine the change of support coefficients for the SMU and panel sized blocks.
4. Transform the $Z^*(V)$ panel estimates to $Y^*(V)$ using the panel anamorphosis function.

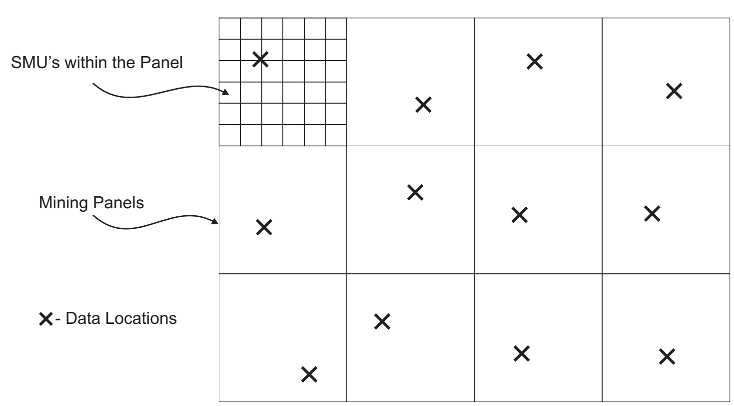

Figure 3.1: Setting for uniform conditioning.

5. Transform the $Z_c(v)$ cutoff grades to $Y_c(v)$ using the SMU anamorphosis function.

6. Calculate the proportion and quantity of metal above each cutoff.

Each step will be covered in the following sections.

3.1 Estimating the Panel Grades

Uniform conditioning relies on a robust estimate of the panel grade. Consider the typical data available during the exploration phase of a mining project. The data are widely spaced and usually conform to a coarse grid. Estimating very small blocks with relation to the data spacing does not produce reliable results. Block kriging a larger mining panel instead of the smaller SMU's will give more reliable results.

Recall the ordinary kriging estimator:

$$z^* (\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha(\mathbf{u})\mathbf{z}(\mathbf{u}_\alpha) \tag{3.1}$$

where n is the number of samples and λ_α is the weight assigned sample α. The weights are determined bys solving the ordinary kriging system of equations:

$$\begin{aligned}
&\sum_{\beta}^{n} \lambda_\beta C\left(\mathbf{u}_\beta, \mathbf{u}_\alpha\right) + \mu\left(\mathbf{u}\right) = C\left(\mathbf{u}, \mathbf{u}_\alpha\right), \quad \alpha = 1, ..., n \\
&\sum_{\beta}^{n} \lambda_\beta = 1
\end{aligned} \tag{3.2}$$

where $C(\mathbf{u}_\beta, \mathbf{u}_\alpha)$ is the covariance between data points β and α, $C(\mathbf{u}, \mathbf{u}_\alpha)$ is the covariance between the point being estimated and data point α, and $\mu(\mathbf{u})$ is the lagrange parameter. Since the kriging algorithm is linear, we can modify the system of equations to estimate the linear average of Z within the block being estimated. By replacing the point-to-point covariance in Equation 3.2 with the point-to-block covariance we get the following equation:

$$\begin{aligned}
\sum_{\beta}^{n} \lambda_\beta C\left(\mathbf{u}_\beta, \mathbf{u}_\alpha\right) + \mu\left(\mathbf{u}\right) &= \overline{C}\left(V\left(\mathbf{u}\right), \mathbf{u}_\alpha\right), \quad \alpha = 1, ..., n \\
&\approx \frac{1}{N}\sum_{j=1}^{N} C\left(\mathbf{u'}_j, \mathbf{u}_\alpha\right), \quad \alpha = 1, ..., n
\end{aligned} \tag{3.3}$$

where V is the panel being estimated and N is the number of discretization points for calculating $\overline{C}(V(\mathbf{u}), \mathbf{u}_\alpha)$.

3.2 Discrete Gaussian Model (DGM)

Data are collected at a very small scale. These small samples are not representative for any scale larger than what they were collected at. Uniform conditioning uses the discrete Gaussian model for the change of support.

The discrete Gaussian model, DGM, is used to permform change of support calculations from the sample scale to a larger block scale. First, an anamorphosis function needs to be fit to the sample data and then the change of support is done. The anamorphosis is a function defined by a polynomial expansion that is fit to the data. Once the polynomials have been fit, the function provides a mapping of the point variable Z to the Gaussian variable Y and vice-versa:

$$\begin{aligned} z(\mathbf{u}) &= \Phi(y(\mathbf{u})) \\ &\approx \sum_{p=0}^{\infty} \phi_p H_p\left[y(\mathbf{u})\right] \end{aligned} \tag{3.4}$$

where ϕ_p is a fitted coefficient for each term of the polynomial expansion, and $H_p\left[Y(\mathbf{u})\right]$ is the hermite polynomial value defined by the term of the expansion and the y value. Equation (3.4) is referred to as the Gaussian anamorphosis. The number of terms in the polynomial expansion is usually limited to a number less than 100. The more terms used, the better the polynomial fitting will be, but the programs will run slower.

Hermite polynomial's are polynomials that are related to the normal distribution. They are defined by Rodrigues' Formula:

$$H_p(y) = \frac{1}{\sqrt{p!} \cdot g(y)} \cdot \frac{d^p g(y)}{dy^p} \tag{3.5}$$

where $g(y)$ is the probabilty of the value y for a standard normal distribution. The first 2 Hermite polynomials, order 0 and 1, are given by:

$$H_0(y) = 1 \qquad\qquad H_1(y) = -y$$

Higher order polynomials can be calculated with the following recursive formula when $p \geq 2$:

$$H_{p+1}(y) = -\frac{1}{\sqrt{p+1}} y H_p(y) - \sqrt{\frac{p}{p+1}} H_{p-1}(y)$$

Hence, it is easy to calculate the hermite polynomials for a given order p, starting from a normal value y.

We now need to fit the anamorposis function to the data by calculating the ϕ coefficients. The first order coefficient is:

$$\begin{aligned} \phi_0 &= E\{\Phi(Y(\mathbf{u}))\} \\ &= E\{Z(\mathbf{u})\} \end{aligned} \tag{3.6}$$

or the expected value of $Z(\mathbf{u})$. Higher order ϕ coefficients are

$$\begin{aligned} \phi_p &= E\{Z(\mathbf{u}) \cdot H_p(Y(\mathbf{u}))\} \\ &= \int \Phi(y(\mathbf{u})) \cdot H_p(y(\mathbf{u})) \cdot g(y(\mathbf{u})) \cdot dy(\mathbf{u}) \end{aligned} \tag{3.7}$$

Equation 3.7 can be approximated with the sample data as a finite summation:

$$\phi_p \approx \sum_{\alpha=2}^{n} (z(\mathbf{u}_{\alpha-1}) - z(\mathbf{u}_\alpha)) \cdot \frac{1}{\sqrt{p}} H_{p-1}(y(\mathbf{u}_\alpha)) \cdot g(y(\mathbf{u}_\alpha)) \tag{3.8}$$

Since there is no correlation between the different polynomials, the variance of the polynomial expansion is:

$$\begin{aligned} Var\{\Phi(Y(\mathbf{u}))\} &= Var\{Z(\mathbf{u})\} \\ &= \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \phi_p\phi_q cov\{H_p(Y(\mathbf{u})), H_q(Y(\mathbf{u}))\} \\ &= \sum_{p=1}^{\infty} \phi_p^2 \end{aligned} \tag{3.9}$$

where $Var\{Z(\mathbf{u})\}$ is the variance of Z at the point support. If the summation is significantly different, the anamorphosis modelling should be checked.

There are two checks that can be done to determine the validity of the modelled anamorphosis function: (1) by comparing the Z to Y transformation function and (2) by comparing the global distribution from the data to the distribution from the anamorphosis. An example of the two plots are shown in Figures 3.2 and 3.3. The experimental anamorphosis is shown as a thin line with the modelled anamorphosis shown as a thick dashed line. The original grade distribution is shown as the bar histogram and the fitted distribution is shown as a line. The experimental and modelled anamorphosis functions should be identical, and the properties of the two distributions should be similar.

The anamorphosis function, Equation 3.4, can be modified to account for the change of support from point data to block data. This is explained in the next section.

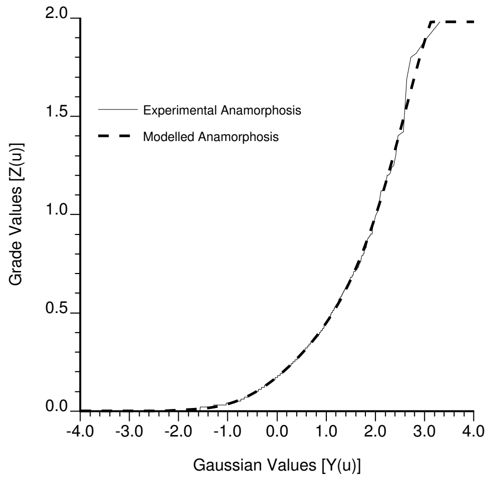

Figure 3.2: Experimental and modelled gaussian anamorphosis.

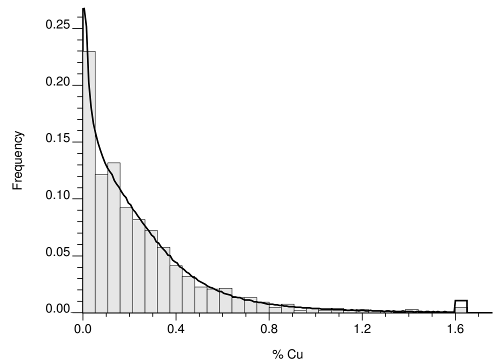

Figure 3.3: Fitted Global Distribution.

3.3 Change of Support

The discrete Gaussian model is used for calculating the change of support for a variable. It controls the shape and variability of the distribution at the larger scale. Recall the point anamorphosis function, equation 3.4:

$$\begin{aligned} Z(\mathbf{u}) &= \Phi(Y(\mathbf{u})) \\ &\approx \sum_{p=0}^{\infty} \phi_p H_p \left[Y(\mathbf{u}) \right] \end{aligned}$$

This function can be modified to account for the change of support from point data to block data by the addition of a change of support coefficient r:

$$\begin{aligned} Z(v) &= \Phi_v(Y(v)) \\ &\approx \sum_{p=0}^{\infty} r^p \phi_p H_p \left[Y(v) \right] \end{aligned}$$

By calculating the value of r, we can determine the distribution of grades for volumes of support larger than the point samples.

The calculation of r requires the variance of the larger support volumes. Typically, there is not enough data available to do this explicitly. The variance of the larger blocks can be estimated using the modelled variogram of the point data. Dispersion variance theory gives us the following relation:

$$\sigma_v^2 = \sigma_{\mathbf{u}}^2 - \bar{\gamma}_{v,v}$$

where v is the smu support volume. The variance of the SMU anamorphosis function is (recall that there is no correlation between the polynomials in the expansion):

$$\begin{aligned} Var\{\Phi_v(Y(\mathbf{u}))\} &= Var\{Z_v(\mathbf{u})\} \\ &= \sigma^2_{\mathbf{u}} - \bar{\gamma}_{v,v} \\ &= \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} r^p \phi_p r^p \phi_q cov\{H_p(Y_v(\mathbf{u})), H_q(Y_v(\mathbf{u}))\} \\ &= \sum_{p=1}^{\infty} r^{2p}\phi_p^2 \end{aligned} \tag{3.10}$$

where $Var\{Z_v\}$ is the variance of Z at the smu support. Since the only unknown parameter is r, its value can be easily calculated so the equalitly is satisfied.

This procedure has to be performed twice. Once to find the change of support for the SMU scale, r, and a second time to find the change of support for the panel scale, r'.

The panel anamorphosis function:

$$\begin{aligned} Z(V) &= \Phi_V(Y(V)) \\ &\approx \sum_{p=0}^{\infty} (r')^p \phi_p H_p\left[Y(V)\right] \end{aligned}$$

The variance of the panel estimates should be calculated directly from the estimates themselves:

$$Var\{Z^*(V)\} = \sigma_V^2$$

Although the panel variance can be estimated by using gammaber values, it is not recommended [15]. The panel change of support coefficient can be calculated by solving for r' in the following equation:

$$Var\{Z^*(V)\} = \sum_{p=1}^{\infty} (r')^{2p}\phi_p^2 \tag{3.11}$$

The correlation between the panel and the smu Guassian transforms is given by:

$$\rho_(v, V) = \frac{r'}{r}$$

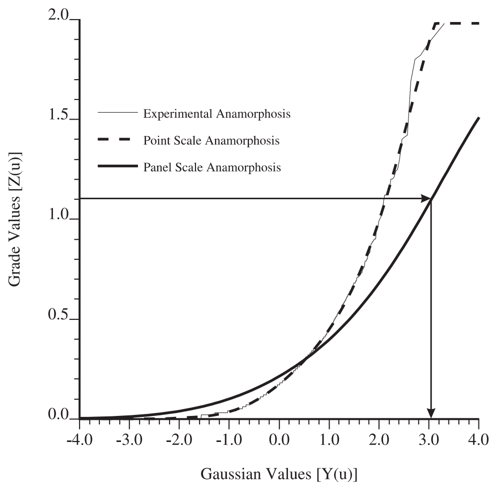

Figure 3.4: Transforming of the panel estimate to gaussian units.

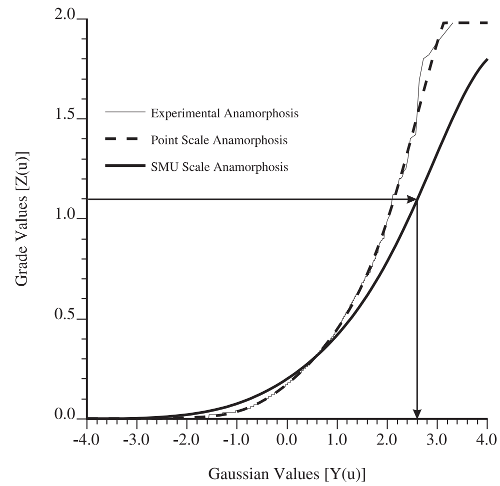

Figure 3.5: Transforming the smu cutoff grade to gaussian units.

3.4 Transform the Panel Estimates to Normal Space

If the panel estimation was done in original grade units, each estimate will need to be transformed to Gaussian space using the panel anamorphosis.

$$\begin{aligned} z(V) &= \Phi_V\left(y(V)\right) \\ y(V) &= \Phi_V^{-1}\left(z(V)\right) \end{aligned} \tag{3.12}$$

See Figure 3.4 for an example.

3.5 Transform the Cutoff Grades to Normal Space

Each cutoff grade needs to be transformed to Gaussian units using the SMU anamorphosis.

$$\begin{aligned} z_c(v) &= \Phi_v\left(y_c(v)\right) \\ y_c(v) &= \Phi_v^{-1}\left(z_c(v)\right) \end{aligned} \tag{3.13}$$

See Figure 3.5 for an example.

3.6 Proportion and Metal Quantity Above Cutoff

Given that we know the panel grade, we can calculate the distribution of the SMU's within that panel. The average of the SMU's is the panel grade, and the variance in Gaussian units is based on the change of support coefficients. For a panel grade, $y(V)$, the SMU's

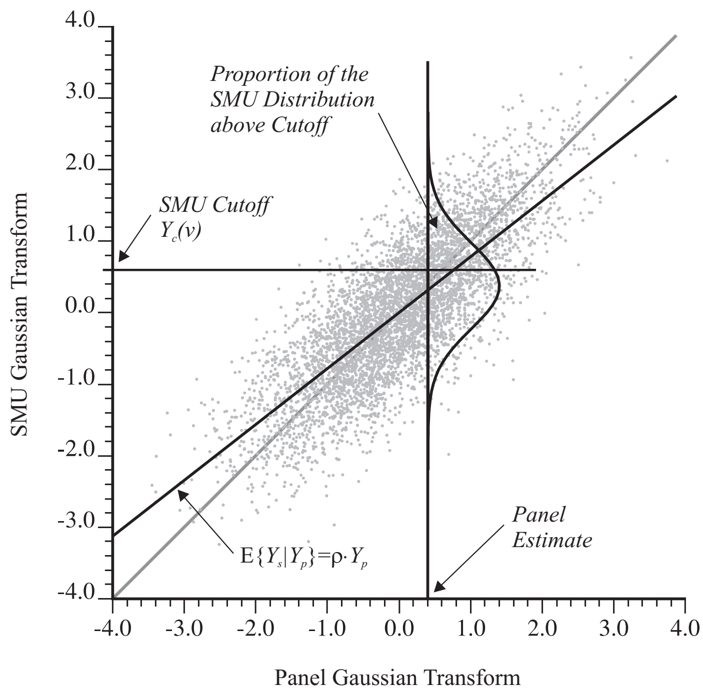

Figure 3.6: Conditional distribution of the smu grades given the panel estimate.

within that panel will have a mean and variance of:

$$
\begin{aligned}
E\left\{y(v)\right\} &= \tfrac{r'}{r} \cdot y(V) \\
Var\left\{y(v)\right\} &= 1 - \left(\tfrac{r'}{r}\right)^2
\end{aligned}
$$

Consider the panel estimate and SMU distribution in Figure 3.6. The recoverable reserves are defined by the proportion and quantity of metal above the cutoff grade. These are easily calculated using the bivariate Gaussian assumption and the SMU anamorphosis function. The conditional expectation line on Figure 3.6 is defined analytically from the bivariate Gaussian distribution between the Gaussian transforms of the SMU and panel grades. The fact that the slope is not 45 degrees is not an indication of conditional bias.

Proportion Above Cutoff

The proportion above the cutoff grade is calculated as follows:

$$
\begin{aligned}
P(z_c) &= P\left[z(v) \geq z_c | z(V)\right] \\
&= P\left[y(v) \geq y_c | y(V)\right]
\end{aligned}
$$

$$= \quad 1 - G\left(\frac{y_c - \left(\frac{r'}{r}\right) y(V)}{\sqrt{1-\left(\frac{r'}{r}\right)^2}}\right) \tag{3.14}$$

Quantity of Metal Above Cutoff

The quantity of metal can be calculated in one of two ways. The first is an integration of the conditional distribution above the cutoff grade [11]:

$$Q(z_c) \quad = \quad \int_{y_c}^{\infty} \Phi_v(y(v)) g(Y(v)|Y(V)) d(y(v)) \tag{3.15}$$

The second is by using the fitted hermite polynomials [14]:

$$\begin{aligned} P(z_c) &= E\left[I_{Z(v)\geq z_c}|Z(V)\right] \\ &= E\left[I_{Y_v \geq y_c}|Y_V\right] \\ &= 1 - G\left(\frac{y_c - \left(\frac{r'}{r}\right) Y_V}{\sqrt{1-\left(\frac{r'}{r}\right)^2}}\right) \end{aligned} \tag{3.16}$$

$$\begin{aligned} Q(z_c) &= E\left[Z(v) I_{Z(v)\geq z_c}|Z(V)\right] \\ &= E\left[\Phi_v(Y(v)) I_{Y(v)\geq y_c}|Y(V)\right] \\ &= \sum_{n=0}^{\infty} q_n \left(\frac{r'}{r}\right)^n H_n(Y(V)) \\ &= \sum_{n=0}^{\infty}\sum_{p=0}^{\infty} \phi_p r^p U_p^n(y_c) \left(\frac{r'}{r}\right)^n H_n(Y(V)) \end{aligned} \tag{3.17}$$

where

$$\begin{aligned} U_0^0(y_c) &= 1 - G(y_c) \\ U_k^0(y_c) &= U_0^k(y_c) = \frac{-1}{\sqrt{k}} H_{k-1}(y_c) g(y_c) \\ U_p^n(y_c) &= \frac{-1}{\sqrt{n}} H_p(y_c) H_{n-1}(y_c) g(y_c) + \sqrt{\frac{p}{n}} U_{p-1}^{n-1} \end{aligned}$$

Grade Above Cutoff

The grade above cutoff is calculated using the estimated quantity of metal and proportion above cutoff:

$$M(z_c) = \frac{Q(z_c)}{P(z_c)}$$

Note that this equation becomes unstable as the proportion of the block above cutoff decreases below 1%.

Chapter 4

Case Studies

The goal of the case studies chapter is to provide step-by-step examples of the uniform conditioning workflow. The first example shows how uniform conditioning is applied to a single panel. This allows the focus of the example to be on the implementation details. The second example is how uniform conditioning can be applied on a larger scale. This allows the calculation of recoverable reserves for a mining bench or the entire deposit.

4.1 Uniform Conditioning for a Single Panel

The following 2-D example is designed to illustrate the implementation details of uniform conditioning. All of the steps that were outlined in Chapter 3 will be covered here.

An entire bench of production data was used for this example. Variogram analysis, anamorphosis modelling, and the change of support were done using the entire data set. Six points were extracted from the data set to estimate the grade for a single panel. A panel size of 60m x 60m was chosen with an SMU size of 10m x 10m.

The histogram of the copper grades is shown in Figure 4.1. The distribution is positively skewed with a mean of 0.25 and a variance of 0.069. Figure 4.2 shows the 6 points extracted from the larger data set and the location of the panel being estimated.

A horizontal omnidirectional variogram was calculated and fit from the data. The modelled variogram is defined as follows and is shown in Figure 4.3:

$$\gamma(\mathbf{h}) = 0.018 + 0.018 \cdot \exp_{a=20}(\mathbf{h}) + 0.033 \cdot sph_{a=325}(\mathbf{h})$$

Using the modelled variogram and the six data points, we can use block kriging to estimate the panel grade. A block discretization of 10x10 was used to estimate the point to

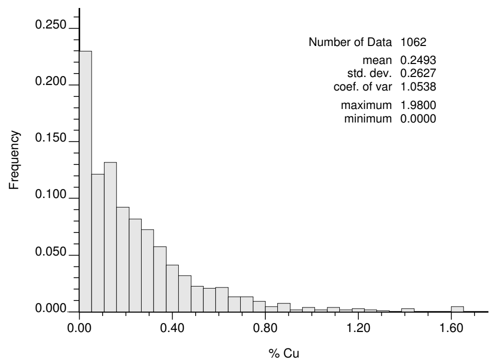

Figure 4.1: Point scale copper histogram.

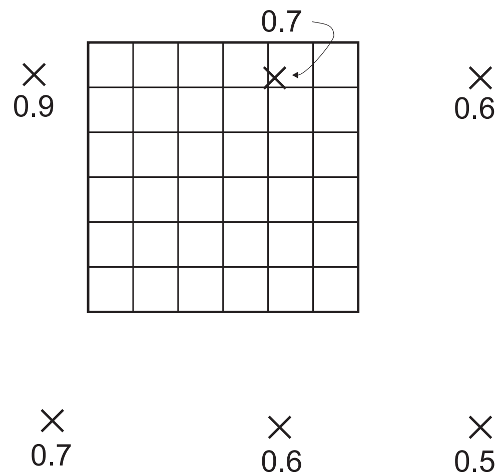

Figure 4.2: Data configuration for estimating the panel grade.

Table 4.1: Fitted hermite polynomial coefficients.

Coefficient	Fitted Value
ϕ_0	0.2493
ϕ_1	-0.2333
ϕ_2	0.1152
ϕ_3	-0.0289
ϕ_4	-0.0007
ϕ_5	0.0084
ϕ_6	-0.0108
ϕ_7	0.0059
ϕ_8	0.0027
ϕ_9	-0.0061
ϕ_{10}	0.0021

block covariance, and the ordinary kriging was used. The estimated panel grade is:

$$z^*(\mathbf{u}) = 0.6775$$

The next step is to fit the grade distribution using hermite polynomials. Eleven polynomials were used to fit the distribution. The values of the fitted coefficients are in Table 4.1. After the coefficients have been calculated, the experimental grade distribution and anamorphosis function can be compared to the fitted model. The fitted distribution is shown in Figure 4.4. Note how similar the experimental and fitted distributions are. Figure 4.5 shows the experimental anamorphosis as a black line and the modelled anamorphosis as a red line. The fitted function is very similar to the experimental

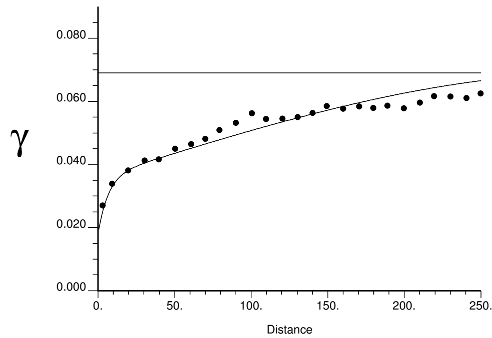

Figure 4.3: Horizontal omnidirectional variogram.

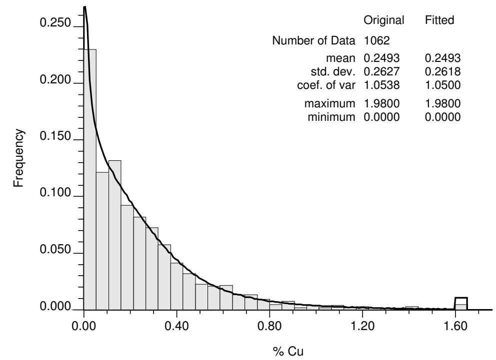

Figure 4.4: Point scale copper histogram with the fitted distribution shown as a line.

We now need to calculate the change of support from the point scale up to the SMU and panel scales. Recall the following equation:

$$\sigma^2_{v,v} = \sigma^2_x - \bar{\gamma}_{v,v}$$

Using the modelled variogram the gammaber values were calculated, along with the variance of the SMU and panels:

$$\bar{\gamma}_{v,v} = 0.028 \qquad \bar{\gamma}_{V,V} = 0.040$$
$$\sigma^2_{v,v} = 0.069 - 0.028 = 0.041 \qquad \sigma^2_{V,V} = 0.069 - 0.040 = 0.029$$

The change of support coefficient for the panels, r', can be calculated by solving:

$$\begin{aligned} Var\{Z(V)\} &= \sum_{n=1}^{nap} \phi_n^2 (r')^{2n} \\ \sum_{n=1}^{nap} \phi_n^2 (r')^{2n} &= 0.029 \\ r' &= 0.67 \end{aligned}$$

Repeat the same procedure for the SMU change of support coefficient, r:

$$\begin{aligned} Var\{Z(v)\} &= \sum_{n=1}^{nap} \phi_n^2 (r)^{2n} \\ \sum_{n=1}^{nap} \phi_n^2 (r)^{2n} &= 0.041 \\ r &= 0.79 \end{aligned}$$

The anamorphosis functions for the SMU and panel blocks are shown in Figure 4.5.

Transform the panel estimate to Gaussian units:

$$\begin{aligned} Y(V) &= \Phi^{-1}(Z(V)) \\ &= \Phi^{-1}(0.68) \\ &= 1.99 \end{aligned}$$

Transform the cutoff grade to Gaussian units:

$$\begin{aligned} Y_c(v) &= \Phi^{-1}(Z_c(v)) \\ &= \Phi^{-1}(0.80) \\ &= 2.02 \end{aligned}$$

Calculate the proportion of the panel above cutoff:

$$\begin{aligned} P(z_c) &= 1 - G\left(\frac{y_c - \left(\frac{r'}{r}\right) Y_V}{\sqrt{1 - \left(\frac{r'}{r}\right)^2}}\right) \\ P(0.8) &= 1 - G\left(\frac{2.02 - \left(\frac{0.67}{0.79}\right) 1.99}{\sqrt{1 - \left(\frac{0.67}{0.79}\right)^2}}\right) \\ &= 1 - 0.72 \\ &= 0.265 \end{aligned}$$

The tonnes above the cutoff are easily calculated using the proportion:

$$T(z_c) = P(z_c) \cdot T_{panel}$$

Calculate the quantity of metal above cutoff:

$$\begin{aligned} Q(z_c) &= \int_{y_c}^{\infty} \Phi_r(Y_v) g(Y_v|Y_V) d(y_v) \\ Q(0.8) &= \int_{2.00}^{\infty} \Phi_r(y_v) g(y_v|2.00) d(y_v) \\ &= 0.252 \end{aligned}$$

$$Q(z_c) = \sum_{n=0}^{nap} \sum_{p=0}^{nap} \phi_p r^p U_p^n(y_c) \left(\frac{r'}{r}\right)^n H_n(Y_V)$$

$$\begin{aligned}Q(0.8) &= \sum_{n=0}^{10}\sum_{p=0}^{10} \phi_p 0.79^p U_p^n(2.00)\left(\frac{0.67}{0.79}\right)^n H_n(1.99)\\ &= 0.252\end{aligned}$$

Calculate the quantity of metal above cutoff:

$$\begin{aligned}Q(z_c) &= \int_{y_c}^{\infty} \Phi_r(Y_v)g(Y_v|Y_V)d(y_v)\\ Q(0.8) &= \int_{2.00}^{\infty} \Phi_r(y_v)g(y_v|2.00)d(y_v)\\ &= 0.252\end{aligned}$$

$$\begin{aligned}Q(z_c) &= \sum_{n=0}^{nap}\sum_{p=0}^{nap} \phi_p r^p U_p^n(y_c)\left(\frac{r'}{r}\right)^n H_n(Y_V)\\ Q(0.8) &= \sum_{n=0}^{10}\sum_{p=0}^{10} \phi_p 0.79^p U_p^n(2.00)\left(\frac{0.67}{0.79}\right)^n H_n(1.99)\\ &= 0.252\end{aligned}$$

Calculate the grade above cutoff:

$$\begin{aligned}M(z_c) &= \frac{Q(z_c)}{P(z_c)}\\ M(0.8) &= \frac{Q(0.8)}{P(0.8)}\\ &= \frac{0.252}{0.265}\\ &= 0.95\end{aligned}$$

A grade proportion curve can be plotted by repeating these calculations for several cutoffs. See Figure 4.6.

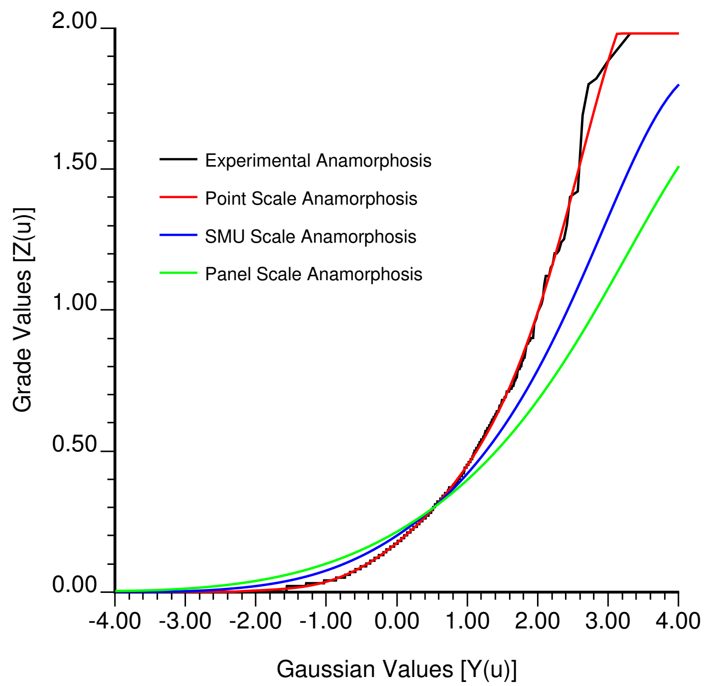

Figure 4.5: Copper anamorphosis function.

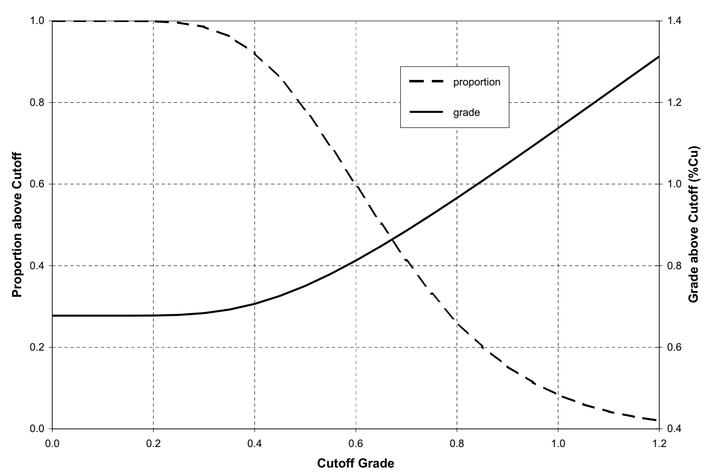

Figure 4.6: Grade-proportion curve for the panel using uniform conditioning.

Chapter 5

Practical Considerations

The theory behind uniform conditioning is straightforward. It is quite reasonable to derive a distribution of SMU grades from a panel grade estimate and a bivariate Gaussian assumption. There are, however, a number of practical considerations. No attempt has been made to undertake and present comparative studies - discussion is limited.

5.1 Place of UC

Uniform conditioning relies on a robust estimation of large panels where the panel estimates are constrained by the surrounding drillhole data. In most cases, the panel size should approximate the drill hole spacing. The panel grade estimates will be quite reliable because of their size relative to the drillhole spacing. The panel estimates will be smooth and free of conditional bias. Local trends and non- stationarities are handled in the robust ordinary kriging of the panel grades.

Indicator kriging, disjunctive kriging, lognormal kriging, and conditional simulation are other methods that can be used for recoverable reserve estimation. Choosing between these different methods can be a difficult task. Practicioners will have to to become familiar with the assumptions and limitations of each method. Most methods can be reliably applied by practitioners with experience with the method. There is an inevitable amount of tradecraft necessary for reliable robust application of any method. There are a number of references with additional information regarding the choice of a non- linear estimation technique, including tests to help determine which method is applicable, see [17].

5.2 Assumptions and Limitations

UC requires several assumptions: (1) that the Gaussian transformed point data, SMU and panels are all bivariate normal, (2) that the change of support model for the SMU's can be

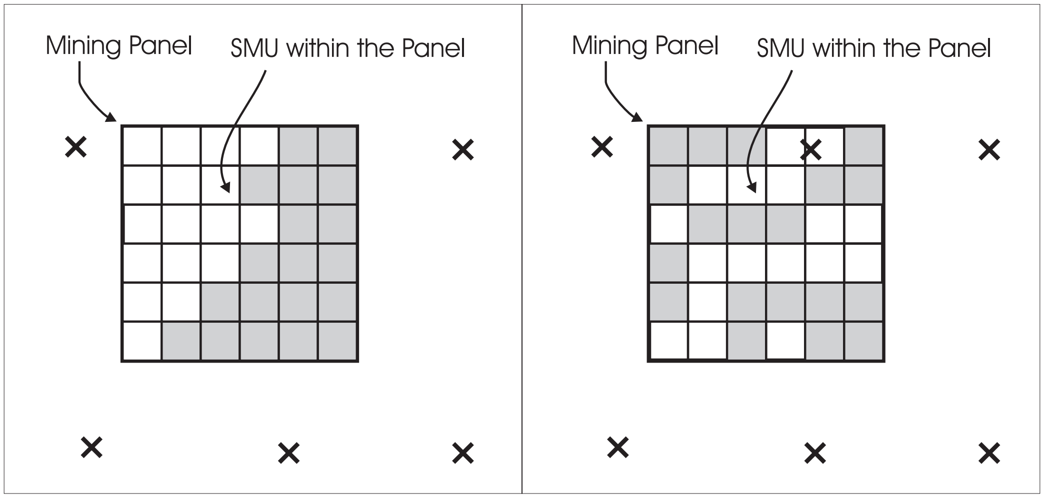

Figure 5.1: Two possible configurations of the SMU's above cutoff.

extended to the panels. These assumptions are required as part of the discrete Gaussian change of support model. These assumptions are no stronger than the assumptions that any of the other common change of support models make (affine or indirect lognormal corrections).

Uniform conditioning does not provide any information regarding where the high or low grade SMU's are within the panel. This is not really a limitation; it is part of the underlying premise of UC - panel grades can be predicted reliably, but SMU grades cannot. Estimation methods that directly estimate smaller blocks would not be considered reliable with widely spaced exploration drilling. The arrangement of ore-grade SMUs within a panel is important, but not prediced by UC. The ore- grade SMUs could be grouped together or scattered throughout the area, see Figure 5.1. Surrounding drillhole data values are used to estimate the panel grade, but they are not used for determining the SMU distribution. For example, consider two panel estimates: one panel is in a homogenous zone where the surrounding samples are all the same, and the other where the surrounding samples are highly variable. When the panel estimates are the same the estimated recoverable reserves are the same. This highlights the need to ensure that uniform conditioning is applied within a stationary domain.

A limitation of UC is that it does not provide a joint distribution of uncertainty between multiple panels and/or multiple SMUs. A big advantage of simulation is that multiple realizations provides the joint distribution ay any scale.

5.3 Change of Support

The recoverable resources/reserves are controlled by the change of support model. The change of support model becomes increasingly important as the panel size becomes large (the information sparse) and the panel estimates become smooth. It may be necessary to adapt the change of support model locally, but the concern is always a lack of local data.

At early stages of exploration, the change of support is controlled by the modelled variogram. Short range variogram structure has a large impact on average variogram and covariance calculations. Sensitivity analysis should be considered for alternative variogram models.

More data will be available as the mine moves into production. The change of support model can be validated against actual production data. By averaging blasthole data to the SMU and panel sizes, the variance of the larger supports can be calculated directly instead of relying on the variogram model. The variogam could be tuned locally to match the variances calculated from upscaled data.

All recoverable reserves calculation methods require a change of support model. The model is made explicit in UC. A similar Gaussian model is embedded in most simulation algorithms.

5.4 Available Software

Isatis has a commercial implementation of uniform conditioning. Isatis is a geostatistics software package written and maintained by Geovariances based in France. They work in partnership with the Centre de Geostatistique at Fontainebleau. The advantage of commerical software is a reasonable user interface, support and a track record of client applications.

The software provided with this guidebook is meant to supplement the commercial alternative. The source code we provide will make it easier for people to understand all of the details and consider modifications to the algorithm.

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> Case study comparing Uniform Conditioning, Indirect lognormal, and the direct lognormal methods for recoverable reserves. No details for uniform conditioning are shown.

[2] J. P. Chilès and P. Delfiner. *Geostatistics: Modeling Spatial Uncertainty.* Wiley-Interscience, New York, 1999.

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[3] C. V. Deutsch and A. G. Journel. *GSLIB: Geostatistical Software Library and User's Guide.* Oxford University Press, New York, second edition, 1998.

> Uniform conditioning is not presented here. The included programs were written to be compatible with GSLIB.

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> This paper presents a different flavour of Uniform Conditioning. The uniform conditioning is done entirely in Gaussian units. It briefly describes global reserve estimation and the Gaussian anamorphosis briefly.

[5] D. Guibal and A. Remacre. Local estimation of the recoverable reserves: Comparing various methods with the reality on a porphyry copper deposit. In G. Verly et al., editors, *Geostatistics for Natural Resources Characterization*, volume 1, pages 435–448. D. Reidel Publishing, Dordrecht, Holland, 1984.

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> This paper presents a case study comparison of Indicator Kriging and Uniform Conditioning. The details of Uniform Conditioning are presented. The results of the comparison are questionable.

[14] J. Rivoirard. *Introduction to Disjunctive Kriging and Non-Linear Geostatistics.* Clarendon Press, Oxford, 1994.

> A fairly complete presentation of non-linear geostatistical methods. Covers the discrete Gaussian model, anamorphosis fitting, and uniform conditioning.

[15] C. Roth and J. Deraisme. The information effect and estimating recoverable reserves. In *Geostats 2000, Geostatistics Congress*, Cape Town, South Africa, April 2000.

> Present the methodology and a case study for incorporating the information effect into reserve estimates using uniform conditioning. This is the only paper found that presents a model for the information effect that is consistent with the discrete Gaussian model.

[16] J. Vann and D. Guibal. Beyond ordinary kriging — an overview of non-linear estimation. In *Beyond Ordinary Kriging: Non-Linear Geostatistical Methods in Practice*, pages 6–25, Perth, Australia, October 30 1998. The Geostatistical Association of Australasia.

> Presents a review of non-linear geostatistical methods over the past 30 years. The methods are presented in an easily understandable format with few equations. Readers are referred to different references for the particulars.

[17] J. Vann, D. Guibal, and M. Harley. Multiple indicator kriging : is it suited to my deposit? In *4th International Mining Geology Conference*, pages 9–17, Coolum, Queensland, Australia, May 14–17 2000. The Australasian Institute of Mining and Metallurgy: Melbourne.

> This paper discusses where Multiple Indicator Kriging is applicable. Uniform Conditioning is considered as an alternative to MIK.

Appendix A

Program Documentation

Several GSLIB compatabile programs were written for implementing unifrom conditioning. These programs are explained here. For details regarding the standard GSLIB program format see [3].

A.1 Fitting the Change of Support Model `pre_uc`

The discrete Gaussian model requires the input histogram be fit by a hermite polynomial expansion. `pre_uc` reads in the data, fits the hermite polynomial expansion, and calculates the change of support coefficients for the smu and panel support sizes. The output file contains the fitted coefficients, ϕ, for the expansion. The change of support coefficients are reported to the screen.

- **datafl:** the input file that contains the grade information and the Gaussian transformed values.
- **varsmu:** variance at the smu support size. The variance is in original units.
- **varpanel:** variance at the panel support size. The variance is in original units.
- **accerror:** acceptable error in the calculation of the change of support coefficients r and r'.
- **np:** the highest order hermite polynomial to use for fitting the anamorphosis function. i.e.: np=3 means that 4 polynomials will be used (0, 1, 2, and 3).
- **outfl:** the output file that contains the fitted hermite polynomial coefficients.

```
                  Parameters for PRE_UC
                  *********************

START OF PARAMETERS:
nscore.trn     -file with input transformation table
0.7            -variance at smu support (calculated with gammabar)
0.4            -variance at panel support (calculated with gammabar)
0.0001         -acceptable error for the r calculation
30             -highest order hermite polynomial to use (0-np)
pre_uc.out     -file for pre_uc output
```

Figure A.1: An example parameter file for `pre_uc`.

A.2 Uniform Conditioning Program `uc`

Uniform conditioning is done after the change of support model has been fitted and the panel grades have been estimated. The panel grades may be estimated using the `kt3d` program from GSLIB or any other kriging program. The uniform conditioning is done using `uc`. Two files are required as input: (1) the file that contains the fitted ϕ, coefficients, and (2) the file that contains the estimated panel grades. The output file contains the proportion, quantity of metal, and grade above the specified cutoff's for each panel that was estimated.

- **anamfl:** the input file that contains the fitted hermite polynomial coefficients.
- **icolhc:** column in the input file that contains the fitted hermite polynomial coefficients.
- **zmin, zmax:** minimum and maximum values for fitting the anamorphosis function.
- **ietype:** units of the panel estimates. 0 if the panel grades have been estimated in the real grade units or 1 if the panels have been estimated in Gaussian units.
- **datafl:** the input file that contains the panel estimates.
- **icolest:** column in the input file that contains the panel estimates.
- **trimming limits:** values outside these limits will not be used.
- **ncut:** the number of cutoff grades to consider.
- **cut(i):** the smu cutoff grades to be considered. In the units of the grade.
- **rsmu** and **rpan:** the change of support coefficients for the smu, r, and panel, r', support sizes.
- **iuctype:** the type of uniform conditioning to perform. 0 to perform unifrom conditioning using the hermite polynomials and 1 to use an integral approximation.

```
                  Parameters for UC
                  *****************

START OF PARAMETERS:
pre_uc.out        -file with the anamorphosis coefficients
 2                -   column with the coefficients
 0.00   4.32      -z minimum and maximum (for extrapolating the anamorphosis)
 0                -estimate type:  0 = Z, 1 = Y
kt3d.out          -file with kriged estimate
 1                -   column with estimate
 0.0   1.0e21     -   trimming limits
 2                -number of cutoff grades
 0.0  0.5         -   cutoff grades
 0.2   0.6        -change of support coefficients for the smu and panel
 1                -uc type:  0 = hermite polynomials, 1 = integral
uc.out            -file for output
 0                -format for the output file:  0=GSLIB, 1=CSV
```

Figure A.2: An example parameter file for `uc`.

- **outfl:** the output file that contains the grade tonnage curve for each panel estimate.
- **iformat:** format for the output file. 0 = GSLIB standard format. 1 = EXCEL CSV format for easy import into EXCEL.

A.3 Upscaling of the Uniform Conditioning Results `post_uc`

The uniform conditioning results can be upscaled to a larger block size. This is useful for calculating the grade tonnage curves for larger areas of the deposit, or the entire deposit. The `post_uc` program upscales the uniform conditioning results to global results. The input file must contain the uniform conditioning output in GSLIB format. The output file will contain the proportion, quantity of metal, and grade above each of the cutoffs in the input file.

- **datafl:** the input file that contains the uniform conditioning output.
- **trimming limits:** values outside these limits will not be used.
- **outfl:** the output file that contains the grade tonnage curve for the upscaled estimates.